1 Fission with improved multiplicity sampling

As an alternative to the fission model described in the previous section there is a modified model that produces more accurate multiplicity distributions for the emitted neutrons and gamma rays from spontaneous and neutron-induced fission. This was motivated by detailed statistical studies of fission chains in multiplying media. This model is data-driven and incorporates all available multiplicity measurements found in the literature. Empirical models are employed whenever multiplicity data are not available. Essentially no data are available for the correlations between the neutrons and gammas, so this model samples these distributions independently. By default, this model effectively scales the multiplicity data to match the average multiplicity value ($\bar{\nu}$) found in the GEANT4 evaluated data library. Therefore, only isotopes that have a measured $\bar{\nu}$ in the data library will emit fission gammas or neutrons. At present the gammas and neutrons are emitted isotropically. The data and empirical models are described in detail in the following subsections.

2 Spontaneous fission and neutron-induced fission

2.1 Neutron number distribution

Based on reasonable assumptions about the distribution of excitation energy among fission fragments, Terrell [1] showed that the probability P_ν of observing ν neutrons from fission can be approximated by a Gaussian-like distribution

$$\sum_{n=0}^{\nu} P_n = \frac{1}{2\pi} \int_{-\infty}^{\frac{\nu-\bar{\nu}+\frac{1}{2}+b}{\sigma}} e^{-\frac{t^2}{2}} dt \tag{1}$$

where $\bar{\nu}$ is the average number of neutrons, σ (set to 1.079) is the width of the distribution, and b is a small correction factor ($b < 0.01$) that ensures that the discrete probability distribution has the correct average $\bar{\nu}$. This model is used when no explicit multiplicity data are available.

Neutron-induced fission data

Zucker and Holden [3] measured the neutron multiplicity distributions for ^{235}U, ^{238}U, and ^{239}Pu (see Tables 1-3), as a function of the incident neutron energy E_n from zero through ten MeV in increments of one MeV. Fig. 1 shows the neutron number distribution for induced fission of ^{235}U. Gwin, Spencer and Ingle [4] measured the distribution at thermal energies for ^{235}U. In addition, there are many measurements of $\bar{\nu}$, the average number of emitted neutrons, for many isotopes. Since there are multiple methods for parameterizing the multiplicity data and renormalizing the overall distributions to agree with the specific measured values of $\bar{\nu}$, we provide four options for generating neutron multiplicity distributions.

The first option uses a fit to the Zucker and Holden data [3] by Valentine [5, 6]. Valentine expressed the P_ν's (for $\nu = 0, ..., 8$) as 5^{th} order polynomials in E_n, the incident neutron energy. These functions $P_\nu(E_n)$ are used to sample the neutron multiplicity for E_n in the range 0 to 10 MeV. When E_n is greater than 10 MeV, E_n=10 MeV is used to generate P_ν.

In addition to using the Zucker and Holden data above for incident neutron energies E_n above 1 MeV, the second option also uses the Gwin, Spencer and Ingle data [4] for ^{235}U at thermal energies (0 MeV) to generate $P_\nu(E_n)$ polynomials. As in the first option, when E_n is greater than 10 MeV, E_n=10 MeV is used to generate P_ν.

The third option implements an alternative polynomial fit from Valentine [6] of P_ν as a function of $\bar{\nu}$ instead of E_n. When a neutron induces a fission, the algorithm converts the incident neutron energy E_n into $\bar{\nu}$ using conversion tables (typically ENDF/EDNL), generates the P_ν distributions for that value of $\bar{\nu}$, and then samples the P_ν distributions to determine ν. Following a suggestion of Frehaut [7], the least-square

E_n	ν=0	1	2	3	4	5	6	7	$\bar{\nu}$
0	.0317223	.1717071	.3361991	.3039695	.1269459	.0266793	.0026322	.0001449	2.4140000
1	.0237898	.1555525	.3216515	.3150433	.1444732	.0356013	.0034339	.0004546	2.5236700
2	.0183989	.1384891	.3062123	.3217566	.1628673	.0455972	.0055694	.0011093	2.6368200
3	.0141460	.1194839	.2883075	.3266568	.1836014	.0569113	.0089426	.0019504	2.7623400
4	.0115208	.1032624	.2716849	.3283426	.2021206	.0674456	.0128924	.0027307	2.8738400
5	.0078498	.0802010	.2456595	.3308175	.2291646	.0836912	.0187016	.0039148	3.0386999
6	.0046272	.0563321	.2132296	.3290407	.2599806	.1045974	.0265604	.0056322	3.2316099
7	.0024659	.0360957	.1788634	.3210507	.2892537	.1282576	.0360887	.0079244	3.4272800
8	.0012702	.0216090	.1472227	.3083032	.3123950	.1522540	.0462449	.0107009	3.6041900
9	.0007288	.0134879	.1231200	.2949390	.3258251	.1731879	.0551737	.0135376	3.7395900
10	.0004373	.0080115	.1002329	.2779283	.3342611	.1966100	.0650090	.0175099	3.8749800

Table 1: Neutron number distribution for induced fission in ^{235}U.

E_n	ν=0	1	2	3	4	5	6	7	8	$\bar{\nu}$
0	.0396484	.2529541	.2939544	.2644470	.1111758	.0312261	.0059347	.0005436	.0001158	2.2753781
1	.0299076	.2043215	.2995886	.2914889	.1301480	.0363119	.0073638	.0006947	.0001751	2.4305631
2	.0226651	.1624020	.2957263	.3119098	.1528786	.0434233	.0097473	.0009318	.0003159	2.5857481
3	.0170253	.1272992	.2840540	.3260192	.1779579	.0526575	.0130997	.0013467	.0005405	2.7409331
4	.0124932	.0984797	.2661875	.3344938	.2040116	.0640468	.0173837	.0020308	.0008730	2.8961181
5	.0088167	.0751744	.2436570	.3379711	.2297901	.0775971	.0225619	.0030689	.0013626	3.0513031
6	.0058736	.0565985	.2179252	.3368863	.2541575	.0933127	.0286200	.0045431	.0031316	3.2064881
7	.0035997	.0420460	.1904095	.3314575	.2760413	.1112075	.0355683	.0065387	.0031316	3.3616731
8	.0019495	.0309087	.1625055	.3217392	.2943792	.1313074	.0434347	.0091474	.0046284	3.5168581
9	.0008767	.0226587	.1356058	.3076919	.3080816	.1536446	.0522549	.0124682	.0067176	3.6720432
10	.0003271	.0168184	.1111114	.2892434	.3160166	.1782484	.0620617	.0166066	.0095665	3.8272281

Table 2: Neutron number distribution for induced fission in ^{238}U.

fits to the ^{235}U data are used for both ^{235}U and ^{233}U neutron induced fission, the fits to ^{238}U are used for ^{232}U, ^{234}U, ^{236}U and ^{238}U, while the fits to ^{239}Pu are used for ^{239}Pu and ^{241}Pu. Data come from Zucker and Holden. For ^{235}U, data comes from Zucker and Holden for E_n greater than 1 MeV, and Gwin, Spencer and Ingle for 0 MeV. The fits are only used when $\bar{\nu}$ is in the range of the $\bar{\nu}$'s for the tabulated data. Otherwise, Terrell's approximation is used.

The fourth option, which is the default, is similar to the third option except that the P_ν distributions are not functions of $\bar{\nu}$, but are left intact as multiplicity distributions for the data listed in Gwin, Spencer and Ingle, and for the data listed in Zucker and Holden. The multiplicity distribution P_ν from which the number of neutrons will be sampled is selected based on the value of $\bar{\nu}$ for a given induced fission event. For instance, if $P_\nu(1MeV)$ has $\bar{\nu} = 2.4$, $P_\nu(2MeV)$ has $\bar{\nu} = 2.6$, and $\bar{\nu}$ is 2.45 at the energy of the incident fission-inducing neutron (this value $\bar{\nu}$ comes typically from cross-section data libraries such and ENDF/ENDL), the probability of sampling the number of neutrons ν from $P_\nu(1MeV)$ and $P_\nu(2MeV)$ will be 75% and 25%, respectively. This technique is only used when $\bar{\nu}$ is in the range of the $\bar{\nu}$'s for the tabulated data. Otherwise, Terrell's approximation is used. This last way of computing ν has several advantages: first, the data as listed in the original papers is used exactly, as opposed to approximated by low-ordered polynomials least-square fitting the original data. Second, the data from the Gwin, Spencer and Ingle paper, and the data from the Zucker and Holden paper is entered as-is as a table in the code, and can easily be checked and maintained if necessary by the application developer. Third the method provides a simple and statistically correct mechanism of sampling the data tables.

E_n	ν=0	1	2	3	4	5	6	7	8	$\bar{\nu}$
0	.0108826	.0994916	.2748898	.3269196	.2046061	.0726834	.0097282	.0006301	.0001685	2.8760000
1	.0084842	.0790030	.2536175	.3289870	.2328111	.0800161	.0155581	.0011760	.0003469	3.0088800
2	.0062555	.0611921	.2265608	.3260637	.2588354	.0956070	.0224705	.0025946	.0005205	3.1628300
3	.0045860	.0477879	.1983002	.3184667	.2792811	.1158950	.0301128	.0048471	.0007233	3.3167800
4	.0032908	.0374390	.1704196	.3071862	.2948565	.1392594	.0386738	.0078701	.0010046	3.4707300
5	.0022750	.0291416	.1437645	.2928006	.3063902	.1641647	.0484343	.0116151	.0014149	3.6246800
6	.0014893	.0222369	.1190439	.2756297	.3144908	.1892897	.0597353	.0160828	.0029917	3.7786300
7	.0009061	.0163528	.0968110	.2558524	.3194566	.2134888	.0729739	.0213339	.0020017	3.9325800
8	.0004647	.0113283	.0775201	.2335926	.3213289	.2356614	.0886183	.0274895	.0039531	4.0865300
9	.0002800	.0071460	.0615577	.2089810	.3200121	.2545846	.1072344	.0347255	.0054786	4.2404900
10	.0002064	.0038856	.0492548	.1822078	.3154159	.2687282	.1295143	.0432654	.0075217	4.3944400

Table 3: Neutron number distribution for induced fission in ^{239}Pu.

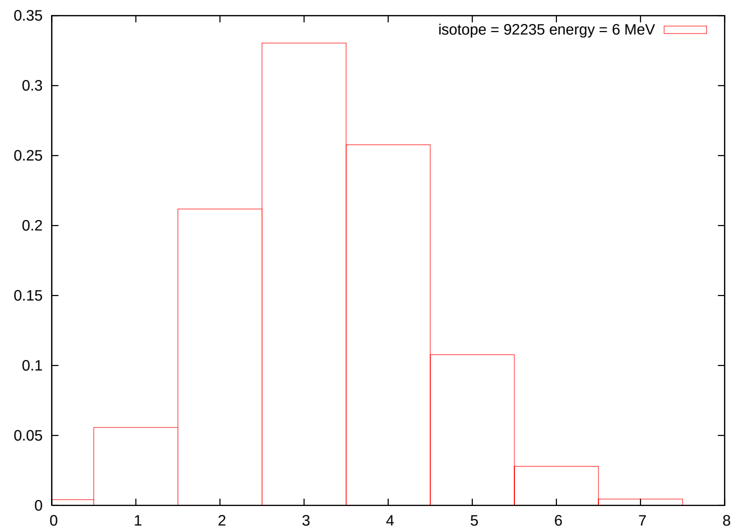

Figure 1: Induced fission in ^{235}U, incident neutron energy = 6MeV

Spontaneous fission data

Table 4 summarizes the spontaneous fission neutron number distributions for several isotopes, along with their references [8, 9, 10, 11, 12, 13]. For ^{252}Cf, the fission module can be set to use either the measurements by Spencer [14], which is the default, or Boldeman [15]. For ^{246}Cm, ^{248}Cm, ^{246}Cf, ^{250}Cf, ^{254}Cf, ^{257}Fm and ^{252}No, the Watt parameters are not available, so even though the number of spontaneous fission neutrons could be sampled, no energy could be attributed to them, and the fission library module thus fails for these 7 nuclides.

If no full multiplicity distribution data exists, the fission module uses Terrell [1]'s approximation with $\bar{\nu}$ from Ensslin [16]. The measured values from Ensslin are listed in Table 5.

2.2 Neutron energy distribution

All of the fission spectra in the Evaluated Nuclear Data Library, ENDL [17] are defined by a simple analytical function, a Watt spectrum defined as

isotope	ν=0	1	2	3	4	5	6	7	8	9
^{238}U [8]	.0481677	.2485215	.4253044	.2284094	.0423438	.0072533	0	0	0	0
^{236}Pu [9]	.0802878	.2126177	.3773740	.2345049	.0750387	.0201770	0	0	0	0
^{238}Pu [9]	.0562929	.2106764	.3797428	.2224395	.1046818	.0261665	0	0	0	0
^{240}Pu [8]	.0631852	.2319644	.3333230	.2528207	.0986461	.0180199	.0020406	0	0	0
^{242}Pu [8]	.0679423	.2293159	.3341228	.2475507	.0996922	.0182398	.0031364	0	0	0
^{242}Cm [8]	.0212550	.1467407	.3267531	.3268277	.1375090	.0373815	.0025912	.0007551	.0001867	0
^{244}Cm [8]	.0150050	.1161725	.2998427	.3331614	.1837748	.0429780	.0087914	.0002744	0	0
^{246}Cm [10]	.0152182	.0762769	.2627039	.3449236	.2180653	.0755895	.0072227	0	0	0
^{248}Cm [10]	.0067352	.0596495	.2205536	.3509030	.2543767	.0893555	.0167386	.0016888	0	0
^{246}Cf [11]	.0005084	.1135987	.2345989	.2742853	.2208697	.1259660	.0301731	0	0	0
^{250}Cf [12]	.0038191	.0365432	.1673371	.2945302	.2982732	.1451396	.0472215	.0040174	.0031188	0
^{252}Cf [14]	.00211	.02467	.12290	.27144	.30763	.18770	.06770	.01406	.00167	.0001
^{252}Cf [15]	.00209	.02621	.12620	.27520	.30180	.18460	.06680	.01500	.00210	0
^{254}Cf [12]	.0001979	.0190236	.1126406	.2638883	.3183439	.1941768	.0745282	.0150039	.0021968	0
^{257}Fm [12]	.0205736	.0520335	.1172580	.1997003	.2627898	.2007776	.1061661	.0333033	.0073979	0
^{252}No [13]	.0569148	.0576845	.0924873	.1437439	.1832482	.1831510	.1455905	.0962973	.0382048	.0026776

Table 4: Neutron number distributions for spontaneous fission, along with their references.

isotope	$\bar{\nu}$	a [MeV^{-1}]	b [MeV^{-1}]	isotope	$\bar{\nu}$	a [MeV^{-1}]	b [MeV^{-1}]
^{232}Th	2.14	1.25	4.0	^{239}Pu	2.16	1.12963	3.80269
^{232}U	1.71	1.12082	3.72278	^{240}Pu	2.156	1.25797	4.68927
^{233}U	1.76	1.16986	4.03210	^{241}Pu	2.25	1.18698	4.15150
^{234}U	1.81	1.29661	4.92449	^{242}Pu	2.145	1.22078	4.36668
^{235}U	1.86	1.29080	4.85231	^{241}Am	3.22	1.07179	3.46195
^{236}U	1.91	1.36024	5.35746	^{242}Cm	2.54	1.12695	3.89176
^{238}U	2.01	1.54245	6.81057	^{244}Cm	2.72	1.10801	3.72033
^{237}Np	2.05	1.19985	4.24147	^{249}Bk	3.40	1.12198	3.79405
^{238}Pu	2.21	1.17948	4.16933	^{252}Cf	3.757	0.847458	1.03419

Table 5: Average number of neutrons per fission and Watt parameters for spontaneous fission [16].

$$W(a,b,E') = Ce^{-aE'} sinh(\sqrt{bE'}) \tag{2}$$

where $C = \sqrt{\pi \frac{b}{4a} \frac{e^{\frac{b}{4a}}}{a}}$, and E' is the secondary neutron energy. The coefficients a and b vary weakly from one isotope to another.

For spontaneous fission, the parameters a and b are taken from Ensslin [16] and are listed in Table 5. For spontaneous fission of ^{236}Pu, there is no data for the Watt fission spectrum. We made the assumption that ^{236}Pu has the same Watt fission spectrum as ^{237}Np since they have approximately the same $\bar{\nu}$ (2.07 versus 2.05). We think this is a good approximation since Cullen [19] showed that the Watt fission spectra for neutron-induced fissions can very well be approximated with the single parameter a by setting b equal to 1.0, instead of the 2 parameters a and b. Since there is only 1 parameter characterizing a Watt spectrum, Watt spectra with identical $\bar{\nu}$'s must have the same value for that parameter a (that is because the integral of the spectrum with respect to the energy gives $\bar{\nu}$, within a normalization factor). If we assume that Watt spectra can be approximated by a single parameter a for spontaneous fissions as well (which we verified and seems to be a valid assumption), there can only be a single Watt spectrum for a given spontaneous fission $\bar{\nu}$. We thus concluded that the Watt spectrum for ^{236}Pu should be close to the Watt spectrum for ^{237}Np and used the Watt parameters of ^{237}Np for ^{236}Pu. The Watt spectrum is used for all isotopes except ^{252}Cf, for which a special treatment summarized by Valentine [6] is applied. The neutron spectrum for ^{252}Cf is sampled from the Mannhart [21] corrected Maxwellian distribution, the Madland and Nix [22] or the Watt fission spectra from Froehner [23]. The Mannhart distribution is used by default.

For neutron-induced fission, the coefficients a and b in Eq. 2 not only vary weakly from one isotope to another, but they also vary weakly with the incident neutron energy E. The fission module follows TART [18, 19]'s implementation by setting the coefficient b equal to 1.0, and using the following functional form for the coefficient $a(E)$:

$$a(E) = a_0 + a_1 E + a_2 E^2 \tag{3}$$

where E is the incident neutron energy.

The coefficients a_0, a_1 and a_2 in Eq. 3 are listed in Table 6 for 40 isotopes. The fission module does not support neutron-induced fission for isotopes other than the ones in the table. The fissioning isotope and incident neutron energy determine the value of the coefficient a in Eq. 2, and the energy E' of the secondary neutron emitted is sampled using the Los Alamos' Monte Carlo sampler attributed to Mal Kalos [20].

isotope	a_2 [MeV^{-3}]	a_1 [MeV^{-2}]	a_0 [MeV^{-1}]	isotope	a_2 [MeV^{-3}]	a_1 [MeV^{-2}]	a_0 [MeV^{-1}]
^{231}Th	6.00949e-05	-0.00836695	0.950939	^{239}Pu	8.50642e-05	-0.0101099	0.887305
^{232}Th	6.54348e-05	-0.00886574	0.955404	^{240}Pu	9.10537e-05	-0.0105303	0.889439
^{233}Th	7.08174e-05	-0.00922676	0.950088	^{241}Pu	9.43014e-05	-0.0107134	0.882632
^{233}Pa	6.35839e-05	-0.00863646	0.924584	^{242}Pu	0.000102656	-0.0113155	0.891617
^{232}U	2.12325e-05	-0.00827743	0.918556	^{243}Pu	0.000106118	-0.0114972	0.885182
^{233}U	6.21336e-05	-0.00845652	0.914717	^{241}Am	9.08474e-05	-0.0104296	0.871943
^{234}U	6.81386e-05	-0.00899142	0.921955	^{242}Am	9.35633e-05	-0.0105612	0.86393
^{235}U	7.32627e-05	-0.00936909	0.920108	^{243}Am	0.00010194	-0.0111574	0.873153
^{236}U	8.06505e-05	-0.00995417	0.92789	^{242}Cm	9.19501e-05	-0.0104229	0.858682
^{237}U	8.33208e-05	-0.0101073	0.917692	^{243}Cm	9.42992e-05	-0.0105099	0.849104
^{238}U	8.96945e-05	-0.0106491	0.925496	^{244}Cm	0.000102747	-0.0111371	0.860434
^{239}U	9.44608e-05	-0.010894	0.917796	^{245}Cm	0.000105025	-0.0112139	0.851102
^{240}U	0.000101396	-0.0115098	0.929395	^{246}Cm	0.00011413	-0.0118692	0.862838
^{235}Np	6.8111e-05	-0.00891619	0.900048	^{247}Cm	0.000115164	-0.0118554	0.851307
^{236}Np	7.21126e-05	-0.00920179	0.895723	^{248}Cm	0.000127169	-0.0127033	0.868624
^{237}Np	7.82371e-05	-0.00967051	0.899575	^{249}Bk	0.000124195	-0.0124047	0.848974
^{238}Np	8.27256e-05	-0.00999353	0.897462	^{249}Cf	0.000112616	-0.0115135	0.819709
^{236}Pu	0.000131389	-0.0080106	0.891084	^{250}Cf	0.000123637	-0.012287	0.835392
^{237}Pu	7.29458e-05	-0.00922415	0.880996	^{251}Cf	0.000122724	-0.0121678	0.82257
^{238}Pu	8.02384e-05	-0.00978291	0.888964	^{252}Cf	0.000133892	-0.0129268	0.837123

Table 6: Values of the a_0, a_1 and a_2 coefficients in Eq. 3 for the neutron induced fission Watt spectrum.

The Watt spectrum for ^{235}U and an incident neutron energy of 6 MeV is shown in Fig. 2.

Neutron energy conservation

The user can choose from three different methods of handling the correlations between neutron energies in a single fission event.

0. (default) Neutron energies are all sampled independently, so there is no explicit energy conservation.

1. A total event energy constraint is imposed in the following way. Beck et al. [24] calculated the average total fission neutron lab kinetic energy as a function of incoming neutron energy E_n for the following 3 isotopes based on the Los Alamos Madland-Nix model [25]:

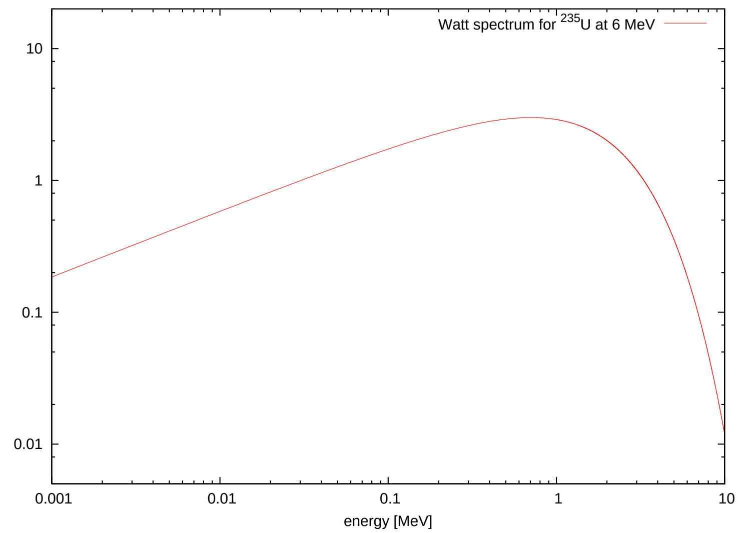

Figure 2: Watt spectrum for ^{235}U and an incident neutron energy of 6 MeV.

$$\begin{aligned} &< E^{tot}_{neutron} >= 4.838 + 0.3004 E_n \quad {}^{235}\mathrm{U} \\ &< E^{tot}_{neutron} >= 4.558 + 0.3070 E_n \quad {}^{238}\mathrm{U} \\ &< E^{tot}_{neutron} >= 6.128 + 0.3428 E_n \quad {}^{239}\mathrm{Pu} \end{aligned} \tag{4}$$

The fission module uses these average values of the kinetic energies as the mean total neutron energy available to the emission of neutrons. For each fission reaction, the number of neutrons N is sampled from the number multiplicity distributions in Sec. 2.1, the total fission neutron energy $E^{tot}_{neutron}$ is sampled from a normal distribution of mean $< E^{tot}_{neutron} >$ and of standard deviation equal to $< E^{tot}_{neutron} > /4$. This normal distribution is truncated at 10 keV to avoid very low total prompt fission neutron energies. The Watt spectrum is then sampled N times to get the energy of these N neutrons. The sampled neutron energies are then rescaled in such a way that the sum of their energies is equal to $E^{tot}_{neutron}$. One of the limitations of this second approach is that it works only for induced fission and for the following 3 isotopes: ^{235}U, ^{238}U and ^{239}Pu.

2. A total event energy constraint is imposed by a method different than that of option 1 above. In 2008, Vogt [26] extended the above Beck et al. [24] method to all actinides, major and minor, in the Evaluated Nuclear Data Library 2008 release, ENDL2008, using data from ENDL2008 and ENDL99. In this extension, the average outgoing prompt gamma energy and prompt neutron energy are expressed by a quadratic expression of the form

$$< E^{tot}_{n/p}(E_n) >= c_{n/p} + b_{n/p} E_n + a_{n/p} E_n^2 \tag{5}$$

where the 3 coefficients are actinide-dependent and the subscripts n and p stand for prompt fission neutrons and gamma-rays. The coefficients of this quadratic form for prompt fission neutrons are given for 73 actinides in table 7. However, because the Watt spectrum is only available for the 40 isotopes listed in table 6, the fission module is limited to these 40 for neutron-induced fission.

Actinide	c_n (MeV)	b_n	a_n (MeV^{-1})	Actinide	c_n (MeV)	b_n	a_n (MeV^{-1})
^{225}Ac	3.478	0.1937	-0.001317	^{239}Pu	6.092	0.3707	-0.002495
^{226}Ac	3.635	0.1231	0.004442	^{240}Pu	5.906	0.2477	0.008608
^{227}Ac	3.396	0.1888	-0.000144	^{241}Pu	6.161	0.2356	0.009310
^{227}Th	4.275	0.1225	0.006569	^{242}Pu	5.926	0.2192	0.008356
^{228}Th	3.787	0.2181	0.003449	^{243}Pu	5.781	0.4692	0.005751
^{229}Th	4.216	0.1339	0.006267	^{244}Pu	5.655	0.2557	0.008807
^{230}Th	3.847	0.1422	0.007380	^{246}Pu	5.145	0.3155	0.007922
^{231}Th	4.095	0.1196	0.006487	^{240}Am	7.150	0.3473	0.002294
^{232}Th	3.401	0.3465	-0.000431	^{241}Am	6.957	0.4243	-0.004504
^{233}Th	3.736	0.2566	0.000663	^{242}Am	7.150	0.3473	0.002294
^{234}Th	3.387	0.2290	0.003476	^{243}Am	7.422	0.3523	-0.002387
^{229}Pa	4.605	0.1744	0.005433	^{244}Am	6.543	0.3837	0.0
^{230}Pa	4.720	0.1879	0.005562	^{240}Cm	7.525	0.2786	0.011040
^{231}Pa	4.524	0.1726	0.006436	^{241}Cm	7.699	0.3648	0.007316
^{232}Pa	4.699	0.1683	0.006763	^{242}Cm	7.701	0.2683	0.011400
^{233}Pa	4.076	0.3671	0.000639	^{243}Cm	8.104	0.2363	0.005492
^{230}U	4.977	0.1832	0.006792	^{244}Cm	7.103	0.2061	0.010830
^{231}U	5.196	0.2127	0.005808	^{245}Cm	7.984	0.2279	0.005426
^{232}U	6.082	0.2782	0.003243	^{246}Cm	6.939	0.2245	0.009390
^{233}U	5.141	0.2540	0.002915	^{247}Cm	8.216	0.3896	0.008595
^{234}U	4.728	0.2339	0.002704	^{248}Cm	7.295	0.2499	0.013550
^{235}U	4.864	0.3114	-0.001424	^{249}Cm	7.124	0.3777	0.008907
^{236}U	4.505	0.2969	0.004555	^{250}Cm	6.973	0.4062	0.006831
^{237}U	4.999	0.2680	0.001783	^{245}Bk	8.210	0.3643	0.009615
^{238}U	4.509	0.3574	-0.004351	^{246}Bk	8.274	0.4764	0.005445
^{239}U	4.580	0.3647	0.004266	^{247}Bk	7.831	0.4266	0.008129
^{240}U	4.561	0.3596	0.000273	^{248}Bk	8.145	0.4796	0.006656
^{241}U	4.268	0.3998	0.002821	^{249}Bk	7.519	0.4021	0.010130
^{234}Np	5.880	0.2311	0.007642	^{250}Bk	7.879	0.4204	0.008308
^{235}Np	5.576	0.2484	0.007751	^{246}Cf	8.900	0.4323	0.009000
^{236}Np	5.080	0.2446	0.008116	^{248}Cf	8.661	0.3877	0.010700
^{237}Np	5.330	0.2768	0.005819	^{249}Cf	9.428	0.4746	0.007067
^{238}Np	5.214	0.2650	0.007559	^{250}Cf	8.226	0.4980	0.007397
^{239}Np	5.416	0.2489	0.004159	^{251}Cf	9.407	0.4454	0.010790
^{236}Pu	6.112	0.2240	0.009279	^{252}Cf	8.627	0.5190	0.007184
^{237}Pu	6.177	0.2599	0.006790	^{253}Cf	8.449	0.2396	0.018650
^{238}Pu	6.087	0.2189	0.008211				

Table 7: Coefficients of Eq. 5 for the energy-dependent average outgoing prompt fission neutron energy.

2.3 Gamma-ray number distribution

The fission module uses Brunson [27]'s double Poisson model for the spontaneous fission gamma ray multiplicity of ^{252}Cf (see Fig. 3).

$$\Pi(G) = 0.682\frac{7.20^G e^{-7.20}}{G!} + 0.318\frac{10.71^G e^{-10.72}}{G!} \tag{6}$$

where G is the gamma ray multiplicity.

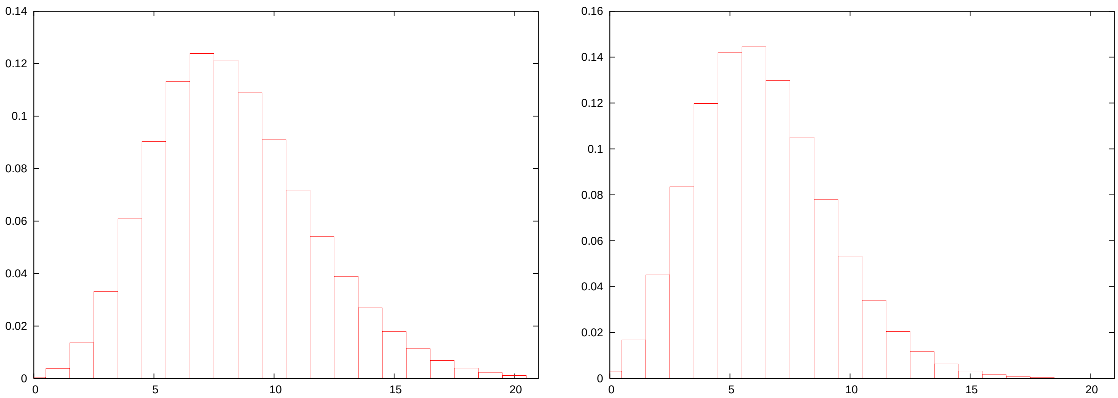

Figure 3: Gamma-ray multiplicity for spontaneous fission of ^{252}Cf (left) and ^{238}U (right).

The prompt gamma ray multiplicity ranges from 0 to 20 gama rays per fission with an average of 8.32 gamma rays per fission. This model is a fit to experimental data measured by Brunson himself.

For other isotopes, there is no data available for the multiplicity of prompt gamma rays. Valentine [28] used an approximation that was adopted by the fission module. The probability of emitting G fission gamma rays obeys the negative binomial distribution:

$$\Pi(G) = \begin{pmatrix} \alpha + G - 1 \\ G \end{pmatrix} p^G (1-p)^G \tag{7}$$

where the parameter p can be written as $p = \frac{\alpha}{\alpha + \bar{G}}$, α is approximately 26 and $\bar{G}$ is the average number of gamma rays per fission. $\bar{G}$ is approximated by

$$\bar{G} = \frac{E_t(\bar{\nu}, Z, A)}{\bar{E}} \tag{8}$$

where

$$E_t(\bar{\nu}, Z, A) = (2.51(\pm 0.01) - 1.13 \cdot 10^{-5}(\pm 7.2 \cdot 10^{-8})Z^2\sqrt{A})\bar{\nu} + 4.0 \tag{9}$$

is the total prompt gamma ray energy, $\bar{\nu}$ is the average number of prompt neutrons, and

$$\bar{E} = -1.33(\pm 0.05) + 119.6(\pm 2.5)\frac{Z^{\frac{1}{3}}}{A} \tag{10}$$

is the average prompt gamma ray energy. The multiplicity distribution for the spontaneous fission of ^{238}U is shown in Fig. 3.

These multiplicity distributions are only estimates and are not measured data. The fission module uses this model for estimating the number of prompt fission gamma rays emitted by both spontaneous and thermal-neutron induced fissions, and also by higher-energy neutron induced fissions. Note that the energy dependence of the gamma multiplicity for neutron induced fission enters through the parameter $\bar{\nu}$, which is calculated by the parent transport code for the specified isotope.

2.4 Gamma-ray energy distribution

The only measured energy spectra for fission gamma rays are from the spontaneous fission of ^{252}Cf and from thermal-neutron-induced fission of ^{235}U. Both spectra are similar [29]. Instead of using either spectra directly, we use the following mathematical representation:

$$N(E) = \begin{cases} 38.13(E - 0.085)e^{1.648E} & E < 0.3 \text{ MeV} \\ 26.8e^{-2.30E} & 0.3 < E < 1.0 \text{ MeV} \\ 8.0e^{-1.10E} & 1.0 < E < 8.0 \text{ MeV} \end{cases} \tag{11}$$

which is shown in Fig. 4. This analytic expression comes from Valentine's [6] and is a fit to the ^{235}U measurements of Maienschein [30, 31] (which are more precise than the ^{252}Cf measurements).

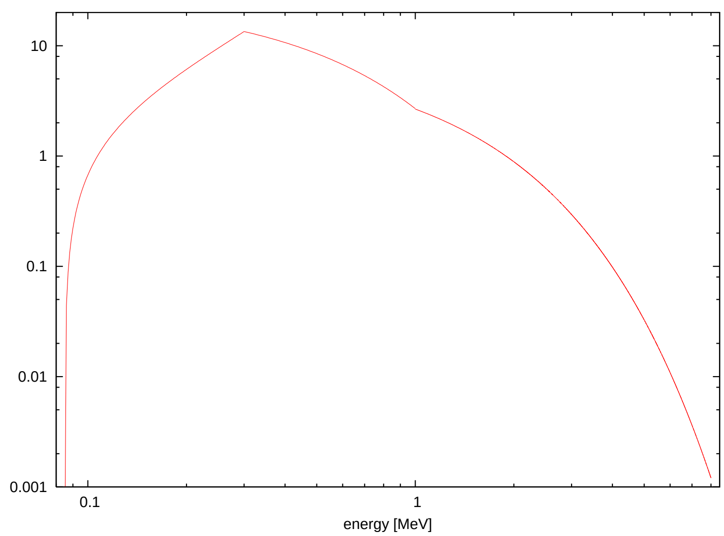

Figure 4: Fission gamma-ray spectrum from fit to ^{235}U measurements.

Gamma-ray energy conservation

The user can choose from three different methods of handling the correlations between gamma-ray energies in a single fission event. The average prompt gamma-ray energy differs significantly between the second and third method given below. This difference is explained in detail in Vogt [26]. The available methods are the same as for neutrons:

0. (default) Gamma-ray energies are all sampled independently from the spectrum shown in Fig. 4, so there is no explicit energy conservation.

1. A total event energy constraint is imposed in the following way. Beck et al. [24] computed the average total fission gamma-ray energy to be:

$$\begin{array}{ll} < E_\gamma^{tot} >= 6.600 + 0.0777E_n & ^{235}\text{U} \\ < E_\gamma^{tot} >= 6.680 + 0.1239E_n & ^{238}\text{U} \\ < E_\gamma^{tot} >= 6.741 + 0.1165E_n - 0.0017E_n^2 & ^{239}\text{Pu} \end{array} \tag{12}$$

For each fission reaction, the number G of prompt fission gammas is sampled from the number multiplicity distributions in Sec. 2.3 using Eqs. 7 through 10, where Eq. 9 giving the total fission gamma-ray energy E_t is replaced by Eq. 12. The gamma-ray spectrum shown in Fig. 4 is then sampled G times to obtain the preliminary energies of the G prompt fission gamma-rays. The total fission gamma-ray energy E_γ^{tot} is sampled from a normal distribution of mean $< E_\gamma^{tot} >$ and standard deviation $< E_\gamma^{tot} > /8$, where $< E_\gamma^{tot} >$ is given by Eq. 12. This normal distribution is truncated at 100 keV to avoid the very low probability region of the prompt fission gamma-ray spectrum shown in Fig. 4. The preliminary prompt fission gamma-ray energies are then rescaled in such a way that the sum of their energies equals E_γ^{tot}. This energy conservation method as well as the one below can give rise to a prompt fission gamma-ray energy spectrum that is different from the one in Fig. 4. One of the limitations of this second approach is that it works only for induced fission and for the following 3 isotopes: ^{235}U, ^{238}U and ^{239}Pu.

2. A total event energy constraint is imposed by a method based on Vogt [26]. This option is very similar to the one above, but instead of using Eq. 12 to determine both the number G of prompt fission gammas and the average outgoing prompt gamma energy $< E_\gamma^{tot} >$, this method uses Eq. 5, where the 3 coefficients are given in table 8. As with the previous option, the total energy $< E_\gamma^{tot} >$ is used to build a normal distribution, which is sampled to obtain the total fission gamma-ray energy E_γ^{tot} available to all G prompt fission gamma-rays. The rescaling of the G preliminary prompt fission gamma-ray energies is identical to the method above. This option applies to all major and minor actinides, but since there is data for just a few few actinides in ENDL, most actinides use a generic set of coefficients.

Actinide	c_p (MeV)	b_p	a_p (MeV^{-1})
^{232}U*	7.256	0.0255	0.000182
^{235}U	7.284	0.2295	-0.00474
^{238}U	6.658	0.01607	-1.22e-7
^{239}Pu	6.857	0.4249	-0.009878
^{252}Cf	6.44186	0.01831	0.
generic	6.95	0.01693	7.238e-8

Table 8: Coefficients of Eq. 5 for the energy-dependent average outgoing prompt fission photon energy. (*) ^{232}U coefficients are used for ^{233}U, ^{234}U, ^{236}U, ^{237}U, ^{240}U and ^{241}U.

2.5 Implementation

For neutron induced fission, this model is intended to be used with the low energy neutron interaction data libraries with class *G4Fisslib* specified in the physics list as the *G4HadronFissionProccess* instead of class *G4NeutronHPFission*. The constructor of *G4FissLib* does two things. First it reads the necessary fission cross-section data in the file located in the directory specified by the environment variable *NeutronHPCrossSections*. It does this by initializing one object of class *G4NeutronHPChannel* per isotope present in the geometry. Second, it registers an instance of *G4FissionLibrary* for each isotope as the model for that reaction/channel. When Geant4 tracks a neutron to a reaction site and the fission library process is selected among all other process for neutron reactions, the method *G4FissLib::ApplyYourself* is called, and one of the fissionable isotopes present at the reaction site is selected. This method in turn calls *G4NeutronHPChannel::ApplyYourself* which calls *G4FissionLibrary::ApplyYourself*, where the induced neutrons and gamma-rays are emitted by sampling the fission library.

For spontaneous fission the user must provide classes *PrimaryGeneratorAction*, *MultipleSource*, *MultipleSourceMessenger*, *SingleSource*, *SponFissIsotope* to generate spontaneous fission neutrons and gammas.

Examples of these classes can be downloaded from `http://nuclear.llnl.gov/simulation`. Spontaneous fissions are generated in the *PrimaryGeneratorAction* class. The spontaneous fission source needs to be described in terms of geometry, isotopic composition and fission strength. Once this information is given, the constructor creates as many spontaneous fission isotopes of class *SponFissIsotope* as specified, and adds them to the source of class *MultipleSource*. When Geant needs to generate particles, it calls the method *PrimaryGeneratorAction::GeneratePrimaries*, which first sets the time of the next fission based on the fission rates entered in the constructor, and then calls the method *MultipleSource::GeneratePrimaryVertex* which determines which one of the spontaneous fission isotopes will fission. This method in turn calls the method *SponFissIsotope::GeneratePrimaryVertex* for the chosen isotope. It is in this method that the neutrons and photons sampled from the fission library are added to the stack of secondary particles. Sources other than spontaneous fission isotopes can be added to the source of class *MultipleSource*. For instance, a background term emitting a large number of background gamma-rays can be added, as long as it derives from the class *SingleSource*. The intensity of that source would be set the same way as for the spontaneous fission isotope sources.

Different sampling methods can be selected by calling the following functions.

void setnudist_(int *nudist)

This selects the data to be sampled for the neutron number distributions for neutron-induced fission. If there is no data available, then in all cases the Terrell approximation is used. The argument *nudist* can take 3 values:

0	Use the fit to the Zucker and Holden tabulated P_ν distributions as a function of energy for ^{235}U, ^{238}U and ^{239}Pu.
1	Use fits to the Zucker and Holden tabulated P_ν distribution as a function of energy for ^{238}U and ^{239}Pu, and a fit to the Zucker and Holden data as well as the Gwin, Spencer and Ingle data (at thermal energies) as a function of energy for ^{235}U.
2	Use the fit to the Zucker and Holden tabulated P_ν distributions as a function of $\bar{\nu}$. The ^{238}U fit is used for the ^{232}U, ^{234}U, ^{236}U and ^{238}U isotopes, the ^{235}U fit for ^{233}U and ^{235}U, the ^{239}Pu fit for ^{239}Pu and ^{241}Pu.
3 (default)	Use the discrete Zucker and Holden tabulated P_ν distributions and corresponding $\bar{\nu}$s. Sampling based on the incident neutron $\bar{\nu}$. The ^{238}U data tables are used for the ^{232}U, ^{234}U, ^{236}U and ^{238}U isotopes, the ^{235}U data for ^{233}U and ^{235}U, the ^{239}Pu data for ^{239}Pu and ^{241}Pu.

void setcf252_(int *ndist, int *neng)

This function is specific to the spontaneous fission of ^{252}Cf. It selects the data to be sampled for the neutron number and energy distributions and takes the following arguments:

ndist: Sample the number of neutrons
- 0 (default) from the tabulated data measured by Spencer
- 1 from Boldeman's data

neng: Sample the spontaneous fission neutron energy
- 0 (default) from Mannhart corrected Maxwellian spectrum
- 1 from Madland-Nix theoretical spectrum
- 2 from the Froehner Watt spectrum

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